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Brane transmutation in supergravity

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Abstract: We study a family of BPS solutions of type-IIA supergravity that can be interpreted as describing the 'transmutation' of a Neveu-Schwarz five-brane into a D4-brane in the presence of a D6-brane. The D4-brane, which terminates on the D6-brane, can be equally well interpreted as a 'pure multipole' configuration of NS5-brane wrapped tightly around the D6-brane. Such a transmutation is a "near-core" version (i.e. near the D6-brane) of the brane-creation that can occur when two branes pass through each other, as in the Hanany-Witten construction. The work below highlights certain charge non-conservation features of type-IIA supergravity.

Keywords: M-Theory, p-branes, D-branes.

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Contents

1. Introduction

It is by now a familiar story that string and M-theory contain various types of branes (see [1, 2, 3, 4] for introductions and reviews) which play a fundamental role in our understanding of the theories. In addition to elucidating the various dualities, they are central to the AdS/CFT correspondence [5, 6] and its generalizations [7] and to matrix theory [8]. Typically, the various branes are a net source of some gauge field, with a total charge that can be measured by surrounding the brane with an appropriate Gauss' law surface and integrating either the field strength or its dual. Measuring the various charges can tell us what sorts of branes are present in a spacetime.

Now, in the associated supergravity theories, not all types of charge are conserved in all cases. As a result, in sufficiently complicated situations, it is possible for one type of brane to 'transmute' into another. The family of type-IIA supergravity solutions investigated below turns out to just such a case, in which the presence of a D6-brane catalyzes the transmutation of an NS5-brane into a D4-brane. This is the supergravity description near the D6-brane core of the sort of brane creation that arises in the Hanany-Witten construction [9] and other examples [10, 11, 12, 13, 14] when two appropriate branes cross. The family of exact supergravity solutions presented below provides a supergravity moduli space in which the dynamics of such processes might be further studied.

Interestingly, in our near-core family the NS5-brane seems to disappear completely despite the fact that the associated current *is* conserved in the setting considered below. What happens is that the NS5-brane hides itself by folding into a "tightly wrapped" configuration that produces no net monopole field. The shape of the resulting 5-brane can be thought of as the limit of a (2-dimensional) paraboloid P crossed with a flat 3-space ($P \times \mathbb{R}^3$) in which the paraboloid degenerates to a half-line. It turns out that the field produced by these NS5-branes does not vanish. However, as no gaussian surface can thread through the (zero-sized) opening of the paraboloid to capture a net flux of field, the net charge of such objects is zero for all practical purposes. Thus, such brane configurations may be called 'fundamental NS5 multipoles:' the field they produce has no monopole part, but contains only the higher multipole moments.[1]

Nonetheless, due to charge non-conservation effects in type-IIA supergravity, our NS5-brane in fact carries a nonzero net D4-brane charge. In the limit in which the paraboloid degenerates to a half-string, the brane may equally well be interpreted as a D4-brane ending on the D6-brane. Now, the family of solutions considered below forms a moduli space that interpolates between a configuration consisting of a flat NS5-brane widely separated from a D6-brane and the configuration with a D4-brane ending on the D6-brane. Thus, by the usual adiabatic arguments, we may consider this family of solutions to represent a *dynamical* process in which a slowly moving NS5-brane approaches from infinity, wraps itself around a D6-brane, and transmutes into a D4-brane ending on the D6-brane.

The family of solutions to be studied below was in fact originally constructed by Hashimoto in [15], using the method of Itzhaki, Tseytlin, and Yankielowicz [16]. It is obtained by considering the 'near-core' solutions for collections of M5-branes intersecting a Kaluza-Klein monopole and performing a Kaluza-Klein reduction. The result must be a collection of D4- and NS5-branes intersecting a D6-brane. By taking into account certain technical subtleties, we uncover the phenomena described above and resolve the 'puzzles' concerning such solutions that were raised in [15].

Another version of the Hanny-Witten process was previously studied [13, 14] using the Born-Infeld worldvolume action to describe the creation of strings as a test D5-brane wraps a D3-brane. Although the back-reaction of the D5-brane on the metric is neglected in such a treatment, the results for the shape of the D5-brane are very similar to ours. The original work [13] produced a description near the D3-brane core and the authors of [14] were able to extend this to a BPS embedding of the D5-brane in the asymptotically flat D3-brane background. If our solutions could be extended to include the asymptotically flat regions, it would be interesting to compare the associated moduli spaces.

We begin the paper with a particularly transparent lower dimensional example in section 2. This example is closely related to the one of central interest but several distracting features have been removed. The construction of the actual type-IIA solutions is reviewed in section 3, correcting few technical points that will be relevant

[1] In curved spacetime, it is not clear that there is a useful distinction between dipole, quadrapole, etc. fields. However, it is certainly meaningful to refer to fields with zero monopole charge as 'pure multipole fields'.

to our discussion. We then proceed in section 4 to map out the relevant branes, verifying that all proceeds in parallel with the lower dimensional case. We conclude with a short discussion in section 5.

2. A transparent example

The set of solutions on which we will focus in sections 3 and 4 will be constructed following [15], and using the method of [16], by first considering an M5-brane in flat spacetime. Now, since the unit-charged Kaluza-Klein monopole is in fact smooth at the center and since one expects that there is in fact a solution[2] representing an M5-brane intersecting an asymptotically flat charge N monopole, the M5-brane in flat space should give the "near-core" version of a solution representing an M5-brane intersecting a unit charged Kaluza-Klein monopole. By the appropriate $\mathbb{Z}_N$ quotient of this solution, one arrives at an M5-brane intersecting an ALE space which represents the near-core solution for a charge N monopole. Kaluza-Klein reduction will then yield the near-core solution for a collection of D4- and NS5-branes intersecting a D6-brane.

In the current section, we consider the simpler case of a membrane in flat $4+1$ Minkowski space, and then reduce it in much the same manner as for the M5-brane mentioned above. The result is a static $3+1$ spacetime, for which the three spatial dimensions are easily visualized. The D6-brane becomes simply a point 0-brane at the origin of the three space. The Kaluza-Klein reduction of the membrane yields a collection of string and membrane charges in the $3+1$ spacetime. Such a lower dimensional example sets the stage well for our discussion in sections 3 and 4 and provides an opportunity to introduce some notation.

The example of the present section is not intended to relate to any particular version of $4+1$ supergravity, though it could certainly be made to do so by considering $4+1$ Minkowski space as some compactification of 11-dimensional supergravity. We simply take the membrane to be a magnetic source (in analogy with the M5-brane to be considered below) of some linear U(1) 1-form field strength $F^{4+1}_{[1]}$. The superscript on the field strength refers to the fact that it represents a $4+1$ field, as opposed to the $3+1$ fields that will arise in the reduction below.

For maximal compatibility with our higher dimensional discussion to come and to fit with the notation of [15], let us refer to the four cartesian spatial coordinates as x_7, x_8, x_9, and x_{10}. These will be grouped into complex combinations:

$$W = x_7 + i x_{10}\,, \qquad V = x_8 + i x_9\,. \tag{2.1}$$

We will use w to denote the magnitude $|W|$ of W, and similarly set $v = |V|$.

[2]The intersection manifold is of sufficiently high dimension, see [17, 18].

We begin with a single two-brane at $V = 0$. Since this is a magnetic source in 4+1 dimensions, its total charge is measured by any 1-sphere which encloses the brane and must be independent of the choice of this 1-sphere. As a result, the symmetries and the Bianchi identity $dF^{4+1} = 0$ for $V \neq 0$ determine the field strength to be of the form

$$F^{4+1}_{[1]} = \frac{Q}{2\pi\,(x_8^2 + x_9^2)}\,(x_8 dx_9 - x_9 dx_8)\,, \tag{2.2}$$

where Q denotes the total charge of the brane.

Let us now introduce three new coordinates (ρ, θ, and ψ) which will describe the reduced spacetime, and a coordinate ϕ which will label points along the Killing orbits. That is, in the coordinates below, we will reduce along the Killing field $k = \partial_\phi$. The particular form of the coordinates is given by

$$\begin{aligned} V &= \rho \sin\frac{\theta}{2}\, e^{i(\psi+\phi)}\,, \\ W &= \rho \cos\frac{\theta}{2}\, e^{i\phi}\,. \end{aligned} \tag{2.3}$$

Note that the Killing field k acts as a simultaneous rotation of both the V and W planes. As a result, the only fixed point is at the origin $V = W = 0$. It may be checked that this is the only singularity in the reduction process. Away from $V = W = 0$, the orbits of the Killing field are circles and smoothly foliate the $4+1$ spacetime. In the type-IIA case of sections 3 and 4, when the gravitational field of the M5-brane is ignored, θ and ϕ become the usual angular coordinates of the D6-brane metric, and the metric reduces to the that of the near-core D6-brane. The D6-brane metric is of course smooth outside the origin. See [7, 15, 16] for details. Thus, we may think of ρ, θ, ϕ as forming a standard set of spherical coordinates on $\mathbb{R}^3$, with some singularity in the origin. In particular, θ ranges over $[0, \pi]$, with $\theta = 0$ and $\theta = \pi$ representing the poles of the 2-sphere. In the current simplified example, this singularity is point-like (a 0-brane). This can also be seen from the 4-dimensional spatial metric

$$|dV|^2 + |dW|^2 = d\rho^2 + \frac{\rho^2}{4}(d\theta^2 + \sin^2\theta d\psi^2) + \rho^2\left(d\phi + \sin^2\frac{\theta}{2} d\psi\right)^2\,, \tag{2.4}$$

where the first three terms define the $\mathbb{R}^3$ described above, and the anomalous $1/4$ factor in front of the 2-sphere metric gives rise to a singularity at $\rho = 0$.

Let us now study the orbits of ∂_ϕ that lie within the membrane. We see that these are at $\theta = 0$ and that there is one orbit for each $\rho > 0$. Note that each orbit lies at a coordinate singularity of ψ, and that this collection of orbits projects under the reduction to the $\theta = 0$ axis. As a result, the projection of the membrane to the reduced spacetime yields a string-shaped charge which terminates at the 0-brane at the origin.

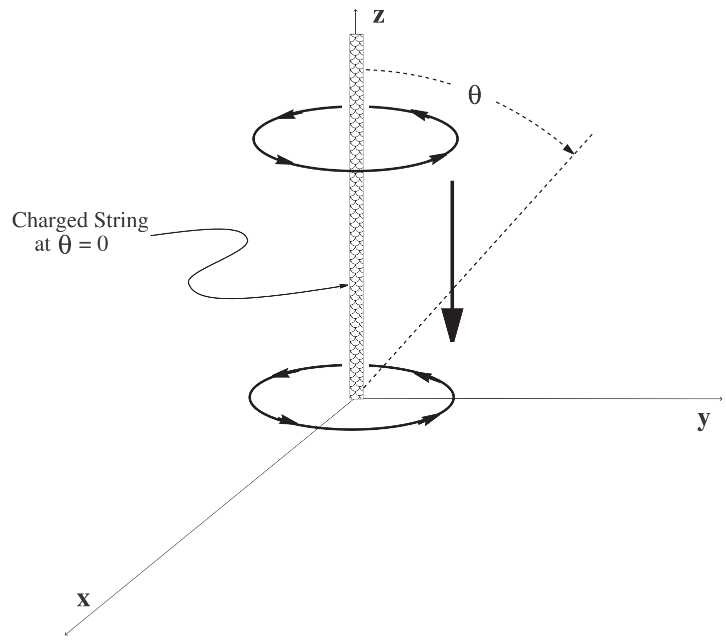

Figure 1: Measuring the charge of the string.

Here we reach a 'puzzle', related to the ones mentioned in [15]. One is tempted to interpret the source along the $\theta = \pi$ axis as the usual charged string of the $3+1$ theory. However, this leads to an immediate question about charge conservation. In $3+1$ dimensions, one measures the charge of a (magnetic) string by integrating a one-form field strength $F_{[1]}$ around a circle enclosing the string. If we consider such a circle that encloses the string, we see that we can easily deform this circle to a point by pulling it down past the origin and shrinking the circle to a point on the negative axis (see figure 1).

A second such puzzle is that Kaluza-Klein reduction of the $4+1$ field strength $F_{[1]}^{4+1}$ yields both a $3+1$ one-form field strength $F_{[1]}$ and a $3+1$ zero-form field strength $F_{[0]}$. The reduction proceeds as

$$F_{[1]}^{4+1} = F_{[1]} + F_{[0]} \wedge d\phi \,. \tag{2.5}$$

For simplicity of notation, we use symbols without superscripts to refer to the $3+1$ dimensional fields. In the present case this yields $F_{[1]} = \frac{Q}{2\pi} d\psi$, $F_{[0]} = \frac{Q}{2\pi}$. The puzzle is that the zero-form field strength $F_{[0]}$ is associated with magnetic *membranes* in $3+1$ dimensions, although none seem to be present here.

For completeness, we mention that there also arises a 2-form field strength $F_{[2]}$ and an associated vector potential $A_{[1]}$ from the Kaluza-Klein reduction of the $4+1$ metric. It turns out that $A_{[1]}$ describes a magnetic monopole:

$$A_{[1]} = \frac{1}{2}(1 - \cos\theta) d\psi \,. \tag{2.6}$$

Let us now return to the issue of charge conservation. Since $F_{[1]}^{4+1}$ is a closed form, it follows from (2.5) that $F_{[1]}$ and $F_{[0]}$ must be closed as well. As a result, string charge defined by integrating $F_{[1]}$ around closed curves should be conserved. That this is in fact the case is clear from the result $F_{[1]} = \frac{Q}{2\pi} d\psi$. Such a field appears to refer to a string that enters from infinity along the $\theta = 0$ axis, passes through the origin, and then exits to infinity along the $\theta = \pi$ axis.

However, a relevant issue is that the field strength $F_{[1]}$ is not gauge invariant. As we can see from (2.5), under a change of coordinates $\phi \to \phi + \gamma(\rho, \theta, \psi)$ (which is just a U(1) gauge transformation of the vector potential A_1 that arises from the Kaluza-Klein reduction), we have $F_{[1]} \to F_{[1]} + F_{[0]} \wedge d\gamma$. As a result, $F_{[1]}$ is a very subtle object in the presence of a non-trivial $A_{[1]}$ bundle, such as that which describes our magnetic monopole. For this reason, one typically adds a term to $F_{[1]}$ to make a gauge invariant field strength, defining

$$\tilde{F}_{[1]} = F_{[1]} - F_{[0]} \wedge A_{[1]} \,. \tag{2.7}$$

Note that this field satisfies

$$d\tilde{F}_{[1]} + F_{[0]} \wedge F_{[2]} = 0 \,, \tag{2.8}$$

in vacuum since $F_{[0]}$ and $F_{[1]}$ are closed.

The importance of all this for our discussion becomes clear when we couple a magnetic current j_1 (representing a string source). The above equation of motion becomes:

$$d\tilde{F}_{[1]} + F_{[0]} \wedge F_{[2]} = {}^*j_1 \,, \tag{2.9}$$

where $*$ represents the Hodge dual. By taking the exterior derivative of this equation, we see that the string current is in fact not conserved.[3] We have

$$d{}^*j_1 = {}^*j_2 \wedge F_{[2]} + F_{[0]} \wedge {}^*j_0, \tag{2.10}$$

where $j_2 = {}^*dF_{[0]}$ is the current associated with membranes and $j_0 = {}^*dF_{[2]}$ is the current associated with the monopole at the origin.

Note that conservation cannot be restored by recognizing that, in analogy with the case for M-branes, D-branes, and fundamental strings [21, 22], certain fields may live on the the monopole and carry the charge of the string. As in [23], one finds that the pullback of certain bulk fields act as sources and sinks of such brane fields, so that charge is still not conserved.

As a result, we see that the string current need not be conserved when the other currents are nonzero. In particular, it is the presence of the monopole (j_0) together with the nonzero $F_{[0]}$ field that allows the string to end at the origin. In contrast, the membrane current j_2 is in fact conserved in this model.

[3] Alternatively, one could define $*j_1 = d\tilde{F}$. Such a current is conserved, but is non-zero (and equal to $-F_{[0]} \wedge F_{[2]}$) even when no branes are present. We prefer the definition (2.9) as it leads to the usual definition of brane charge [19, 2, 20].

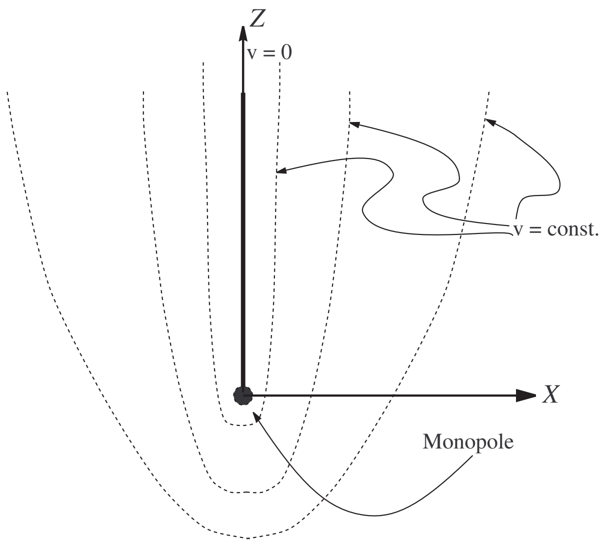

Figure 2: The constant v surfaces. The complete 3-dimensional reduced ALE space is obtained by rotating this planar diagram about the Z-axis.

There remains, however, the issue of the nonzero $F_{[0]}$ field. From whence does it arise? We will see that it arises because the string charge along the $\theta = 0$ axis may equally well be interpreted as a tightly rolled paraboloid of membrane. It turns out that such a tightly rolled membrane is in fact physically equivalent to a half-string.

To see that this is the case, we again follow the lead of [15] and consider the more general family of solutions obtained by placing not a single membrane at $V = 0$ in the $4+1$ spacetime, but in fact a uniform density of branes around the circle $|V| = b$. The branes are still oriented along the W plane, so that we recover the configuration above in the limit $b \to 0$. In this case we have

$$\begin{aligned} F^{4+1}_{[1]} &= \frac{Q}{2\pi}\left(\frac{x_8 dx_9 - x_9 dx_8}{x_8^2 + x_9^2}\right) \qquad && \text{for } |V| > b\,, \\ F^{4+1}_{[1]} &= 0 && \text{for } |V| < b\,. \end{aligned} \tag{2.11}$$

Thus, for $v = \rho \sin\frac{\theta}{2} > b$, the gauge fields are independent of b, while they vanish[4] for $v < b$. For the reader's convenience, a diagram showing the constant v surfaces in the reduced 3-space is shown in figure 2. For later reference we also give the explicit projection from the 4 dimensional space to the 3-dimensional space depicted above:

$$\begin{aligned} X &= \bar{W}V + W\bar{V} = R\sin\theta\cos\psi\,, \\ Y &= i(\bar{W}V - W\bar{V}) = R\sin\theta\sin\psi\,, \\ Z &= |W|^2 - |V|^2 = R\cos\theta\,, \end{aligned} \tag{2.12}$$

where $R = \rho^2/2$.

[4]Except of course $F_{[2]}$, which arises purely from the Kaluza-Klein reduction of the 5-dimensional metric.

We see that there is a discontinuity in $F_{[0]}$ across the surface $v = b$. Such a domain wall is associated with a membrane charge sitting on the paraboloid $v = b$. We may say that the membrane generates this flux of $F_{[0]}$. What is unusual about this case is that the field strength $F_{[0]}$ does not vanish in the $b \to 0$ limit in which the paraboloid degenerates. Instead, the presence of the monopole at the focus of the paraboloid stabilizes the $F_{[0]}$ field. However, in the $b = 0$ limit, one cannot see inside the degenerate paraboloid and there are no longer any discontinuities in the field strength. Thus, the limiting distribution of branes cannot be said to contain a net membrane charge. Instead, we refer to it as a 'pure multipole' configuration of branes. We will see that much the same thing occurs in our higher dimensional case below, where the resulting field strength is more interesting.

So, for $b \neq 0$ it is clear that we have a membrane present on the paraboloid. What about the strings in this case? Note that, since the membrane curves around the monopole, the term ${}^*j_2 \wedge F_{[2]}$ is nonzero. Thus, as noted above, strings will not be conserved in this spacetime. Instead, the membrane will act as a source of strings.[5]

Whether one wishes to consider such strings as 'separate' from the membrane or as a part of the same object is simply a matter of record keeping. The interesting question is whether we may consider *all* strings in the spacetime to be generated by the membrane. The answer to this question is 'yes'. This fact follows from charge (non-)conservation. We simply note that any strings entering the spacetime must do so along the paraboloid in a rotationally symmetric way, converging toward the vertex of the paraboloid. Thus, if charge were conserved, there would need to be some net flux of strings exiting the vertex of the paraboloid. However, this is not the case. All brane sources in the spacetime are confined to the paraboloid itself: Using the gauge invariant field strengths, one can readily check that the source-free equations of motion are satisfied in the region $v \neq b$. Of course, string charge is not in fact conserved, and we therefore conclude that the membrane is configured just so as to annihilate exactly the amount of string charge that enters from infinity.

Running the argument backwards, we could say that we begin with zero string charge at the vertex and that that quantity of string charge exiting to infinity is exactly the amount produced by the membrane. Thus, it is natural to view the membrane as the primary source for the fields, and the strings as a secondary consequence.[6] In this way, we can view the string ending on the monopole as being

[5]On the other hand, since the monopole is inside the paraboloid, we have ${}^*j_0 \wedge F_{[0]} = 0$ and the 0-brane no longer creates or destroys strings.

[6]Admittedly, this viewpoint is adapted to the high symmetry of this situation. If one now adds to our solution the proper small flux of strings coming in along the paraboloid from infinity near $\psi = 0$ and exiting along the paraboloid near $\psi = \pi$ one should be able to, for example, obtain a solution in which the D4-brane charge still vanishes at some point on the paraboloid, though this point will no longer be at the vertex. It would be interesting to know whether such a solution breaks further supersymmetries. One would suspect that it does, since it clearly breaks rotational invariance and supersymmetries square to Killing vectors.

equivalent to the degenerate limit of the paraboloidal membrane. We may call this process the transmutation, catalyzed by the presence of the monopole, of a membrane into a string.

3. The M $\rightarrow$ IIA reduction.

We now review the construction of [15], which produces a distribution of NS5-, D4-, and D6-brane charge in type-IIA supergravity. We proceed quickly, as the present case is in direct parallel with the lower dimensional example presented in section 2.

Consider the metric and gauge field corresponding to a solution containing a $M5$-brane with charge q[24] ,

$$\begin{aligned} ds_{11}^2 &= f^{-1/3}(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_7^2 + dx_{10}^2) + \\ &\quad + f^{2/3}(dx_4^2 + dx_5^2 + dx_6^2 + dx_8^2 + dx_9^2)\,, \end{aligned} \tag{3.1}$$

$$A_{[6]}^{(11)} = \frac{f-1}{f} dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_7 \wedge dx_{10}\,, \tag{3.2}$$

where,

$$f = 1 + \frac{q}{r^3} \qquad \text{and} \qquad r^2 = x_4^2 + x_5^2 + x_6^2 + x_8^2 + x_9^2\,.$$

The field strength associated to $A_{[6]}^{(11)}$ is the 7-form $F_{[7]}^{(11)} = dA_{[6]}^{(11)}$. Its dual is the 4-form $F_{[4]}^{(11)} = {}^*F_{[7]}^{(11)}$ which is straightforwardly computed from (3.1) and (3.2),

$$\begin{aligned} F_{[4]}^{(11)} &= \frac{3q}{r^5}\Big(dx_4 \wedge dx_5 \wedge dx_6 \wedge [x_8 \wedge dx_9 - x_9 \wedge dx_8] + \\ &\quad + [x_4 dx_5 \wedge dx_6 + x_5 dx_6 \wedge dx_4 + x_6 dx_4 \wedge dx_5] \wedge dx_8 \wedge dx_9\Big)\,. \end{aligned} \tag{3.3}$$

At this point the topology and differential structure of the spacetime are those of $\mathbb{R}^{1,10}$. However, we are interested in the topology and differential structure of $\mathbb{R}^{1,6} \times \mathbb{C}^2/\mathbb{Z}_N$, which is obtained as follows. Define as in the preceding section $W = x_7 + ix_{10}$ and $V = x_8 + ix_9$. The $\mathbb{Z}_N$ orbifold group acts by rotating simultaneously this two complex coordinates, i.e.

$$\begin{aligned} \mathbb{Z}_N : \mathbb{C}^2 &\longrightarrow \mathbb{C}^2 \\ (V, W) &\longmapsto e^{2\pi i/N}(V, W)\,. \end{aligned}$$

$\mathbb{C}^2/\mathbb{Z}_N$ is the 4-dimensional manifold defined by identifying points in $\mathbb{C}^2$ which are related by the action of $\mathbb{Z}_N$. Our spacetime manifold is topologically the product of $R^{6,1}$ (in $x_0, \ldots, x_6$) and $\mathbb{C}^2/\mathbb{Z}_N$ (in $x_7, \ldots, x_{10}$). Clearly, all the fields are invariant under rotations in the (x_7, x_{10}) and (x_8, x_9) planes. Therefore defining $W = we^{i\phi/N}$ and $V = ve^{i(\psi+\phi/N)}$, for $\phi, \psi \in [0, 2\pi)$, the vector field ∂_ϕ is a Killing vector field. One dimensionally reduces along this vector field to obtain the desired 10-dimensional solutions.

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Recall that dimensional reduction to 10-dimensional type-IIA supergravity identifies the different fields as follows, where z is a coordinate along the Killing orbits used in the reduction:

$$
\begin{aligned}
ds^2_{(11)} &= e^{-2\Phi/3} ds^2 + e^{4\Phi/3}(dz + A_{[1]})^2 \,, &(3.4)\\
F^{(11)}_{[4]} &= \tilde{F}_{[4]} + F_{[3]} \wedge (dz + A_{[1]}) \,. &(3.5)
\end{aligned}
$$

Here, Φ is the usual dilaton field and ds^2 is the metric in the string frame. It will be useful to define coordinates in analogy with (2.3) through

$$
V = \rho \sin\frac{\theta}{2} e^{\psi + \phi/N} \,, \qquad W = \rho\cos\frac{\theta}{2} e^{\phi/N} \,. \tag{3.6}
$$

Note that in these coordinates, the action of $\mathbb{Z}_N$ takes the convenient form $\phi \longmapsto \phi + 2\pi$. As it is standard in dimensional reduction to use a Killing vector field with dimensions of inverse length, one takes $\partial_z = \partial_\phi / R_{11}$ for some fixed radius R_{11}. To present the dimensionally reduced field strengths it is convenient to first introduce spherical coordinates in $(x_4, x_5, x_6, x_8, x_9)$ defined by

$$
\begin{aligned}
x_4 &= r\cos\alpha \,, & x_5 &= r\sin\alpha\cos\beta \,,\\
x_6 &= r\sin\alpha\sin\beta\cos\gamma \,, & x_7 &= w\cos(\phi/N) \,,\\
x_8 &= r\sin\alpha\sin\beta\sin\gamma\cos(\psi + \phi/N) \,, & x_9 &= r\sin\alpha\sin\beta\sin\gamma\sin(\psi+\phi/N) \,,\\
x_{10} &= w\sin(\phi/N) \,,
\end{aligned} \tag{3.7}
$$

for $0 \le \alpha, \beta, \gamma < \pi$, $0 \le \psi, \phi < 2\pi$ and r, w real and positive. Taking $(r, \alpha, \beta, \gamma, \psi, w)$ to label the Killing orbits leads to the same reduction as (3.6). Now, from (3.5) we obtain,

$$
\begin{aligned}
\tilde{F}_{[4]} &= -\frac{3qw^2\sin^5\alpha\sin^4\beta\sin^3\gamma}{r^2\sin^2\alpha\sin^2\beta\sin^2\gamma\,(1+q/r^3) + w^2}\, d\alpha\wedge d\beta\wedge d\gamma\wedge d\psi \,, &(3.8)\\
F_{[3]} &= -\frac{3q}{NR_{11}}\sin^3\alpha\sin^2\beta\sin\gamma\, d\alpha\wedge d\beta\wedge d\gamma \,, &(3.9)
\end{aligned}
$$

while the 1-form is given by,

$$
A_{[1]} = NR_{11}\frac{r^2\,(1+q/r^3)\sin^2\alpha\sin^2\beta\sin^2\gamma}{w^2 + r^2\,(1+q/r^3)\sin^2\alpha\sin^2\beta\sin^2\gamma} d\psi \,. \tag{3.10}
$$

The ten-dimensional fields above are somewhat different from the fields originally presented in [15]. In particular, our 3-form field strength $F_{[3]}$ is non-zero as given by (3.9). This difference is due to certain subtleties associated with dimensional reduction and Hodge duals in spacetimes with non-diagonal metrics that were not taken fully into account in [15]. In particular, it is the field strength $F^{(11)}_{[4]}$ that is related to the standard ten-dimensional fields by (3.5), while the relation of its Hodge dual 7-form field strength $(F^{(11)}_{[7]})$ to the ten-dimensional fields is more complicated.

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Note that (3.10) represents a monopole located at the origin of the reduced ALE space. Its world-volume spans the entire $\mathbb{R}^{1,6}$ factor, which shows it to be a 6-dimensional magnetic object: a $D6$-brane of type-IIA supergravity. As usual, the magnetic potential shows a string-like singularity, extending from the origin to infinity through the negative Z-axis of the reduced ALE space. This corresponds to the points $w = 0$ in the above coordinates. As in section 2, we may again generalize to a family of solutions constructed from a set of M5-branes oriented along the 5-space $(W, \overline{W}, x_1, x_2, x_3)$ and smeared over the circle $|V| = b$.

4. Brane mapping

Having reviewed the basic construction of the solution in section 3, we need only check a few basic features in order to map out all of the branes and show conclusively that it can be interpreted in parallel with the discussion of section 2. One distracting feature of the present case is that the magnetic monopole associated with the $F_{[2]}$ resulting from the Kaluza-Klein reduction is no longer a point defect, but an entire D6-brane. This makes a discussion of charge conservation more subtle as D4-brane charge (the analogue of string charge in section 2) is now measured not just with a circle around the ψ axis, but with a 5-sphere which extends in certain directions along the D6-brane. As a result, the corresponding 5-sphere can no longer be deformed to a point without intersecting any charge. What happens is that, when one tries to pull the 5-sphere down into the lower half of the ALE space, one necessarily encounters some part of the D6-brane located at some nonzero value of x_4, x_5, or x_6. However, as we will see, this feature is a mere distraction and does not significantly change the interpretation of the solution.

Let us first comment on charge conservation. To clarify this point, note that from (3.5) the equation of motion for $\tilde{F}_{[4]}$ away from any source is

$$d\tilde{F}_{[4]} + F_{[2]} \wedge F_{[3]} = 0\,. \tag{4.1}$$

Here we have used the fact that $dF^{(11)}_{[4]} = 0$ outside the sources, and that $F_{[3]}$ is closed, which is also clear from (3.5). When 4-branes are present, equation (4.1) is written

$$d\tilde{F}_{[4]} + F_{[2]} \wedge F_{[3]} = {}^*j_4 \tag{4.2}$$

in analogy with (2.9), where now ${}^*j_4$ is the Hodge dual of the current associated with 4-brane charge. Taking the exterior derivative of this equation we find, in analogy with (2.10),

$$d^*j_4 = {}^*j_6 \wedge F_{[3]} + F_{[2]} \wedge {}^*j_5\,, \tag{4.3}$$

where j_6 and j_5 are the currents associated to D6- and NS5-branes respectively. This shows that the 4-brane charge need not be conserved in the presence of NS5- and D6-branes, which is indeed the case in which we are interested.[7]

We now check that all of the branes are in fact located on the paraboloid at $v = b$. To do so, recall that for the original 11-dimensional solution, we had $dF^{(11)}_{[4]} = {}^*j^{(11)}_5$. The general case $b \neq 0$ is obtained [15] by including a set of $M5$-branes smeared along ∂_ϕ at $|V| = b$. In this way we keep the original symmetry along ∂_ϕ, and the (11-dimensional) Hodge dual of the current is,

$$*j^{(11)}_5 = \frac{Q}{2\pi}\delta(x_4)\delta(x_5)\delta(x_6)\delta(v-b)dx_4\wedge dx_5\wedge dx_6\wedge dv\wedge d\tilde{\psi}\,,$$

where $\tilde{\psi} = \psi + \phi/N$ is the angle in the (x_8, x_9) plane. From this expression we can easily get the equations of motion for the 10-dimensional fields,

$$d{}^*F_{[3]} = {}^*j_5 = \frac{Q}{2\pi N R_{11}}\delta(x_4)\delta(x_5)\delta(x_6)\delta(v-b)dx_4\wedge dx_5\wedge dx_6\wedge dv\,, \quad (4.4)$$

$$d\tilde{F}_{[4]} + F_{[2]}\wedge F_{[3]} = {}^*j_4 = \frac{Q}{2\pi}\delta(x_4)\delta(x_5)\delta(x_6)\delta(v-b)dx_4\wedge \\ \wedge dx_5\wedge dx_6\wedge dv\wedge\left(d\psi - \frac{A_{[1]}}{NR_{11}}\right). \quad (4.5)$$

In such coordinates $A_{[1]}$ takes the form,

$$A_{[1]} = \frac{NR_{11}}{1 + w^2/fv^2}d\psi\,. \quad (4.6)$$

From (4.5) and (4.6) we immediately see that there can be 4-brane charge only on the surface of the paraboloid at $x_4 = x_5 = x_6 = 0$, $v = b$. Note that ${}^*j_4$ goes to zero at both $v = 0$ and $w = 0$. This rules out the possibility of having a singularity at those points, where the form $d\psi$ is not well defined. The harmonic function f, which also diverges only at the paraboloid, is given by

$$f = 1 + \frac{Q}{16\pi^3}\int_0^{2\pi}\frac{d\mu}{(s^2 + |v - be^{i\mu}|^2)^{3/2}}\,,\,, \quad (4.7)$$

where $s^2 = x_4^2 + x_5^2 + x_6^2$. As a result, it is clear that all branes lie along the paraboloid $v = b$.

The field strengths of the 10-dimensional theory can be obtained from the 11-dimensional solution which, in this generic case is,

$$F^{(11)}_{[4]} = \Big(Av[x_4dx_5\wedge dx_6 + x_5dx_6\wedge dx_4 + x_6dx_4\wedge dx_5]\wedge dv - \\ - Bv^2[dx_4\wedge dx_5\wedge dx_6]\Big)\wedge d\left(\psi + \frac{\phi}{N}\right), \quad (4.8)$$

[7]This is, of course, only a special case of a general subtlety involving charge conservation in supergravity that arises whenever the gauge invariant field strength satisfied an equation of motion of the form (4.1). Thus, similar non-conservation effects hold for M2-brane charge in the presence of M5-brane charge, for strings in the presence of various D-branes, etc.

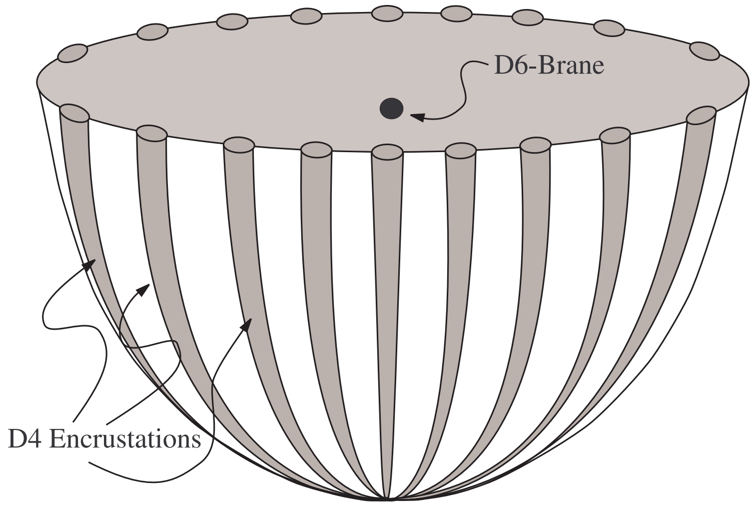

Figure 3: The D4-charge goes to a constant value for large values of w. For $w = 0$, at the vertex of the paraboloid, the charge goes to zero which is possible due to the non conservation of this charge. The individual D4-brane sources are distributed along the paraboloid at values of constant ψ and can be seen as encrustations on the 5-brane.

with

$$\begin{aligned} A &= \frac{-3Q}{16\pi^3}\int_0^{2\pi}\frac{d\mu}{(s^2+v^2+b^2-2bv\cos\mu)^{5/2}}, \\ B &= \frac{-3Q}{16\pi^3}\int_0^{2\pi}\frac{(1-\frac{b}{v}\cos\mu)d\mu}{(s^2+v^2+b^2-2bv\cos\mu)^{5/2}}. \end{aligned} \tag{4.9}$$

Now, for non-zero b, we may easily work around the distraction of the D6-brane in interpreting the D4-brane charge distribution. By symmetry, any D4-branes on the paraboloid must enter in a rotationally invariant fashion along the opening at infinity and run along surfaces of constant ψ. Consider then the subsurface S at some constant ψ, $v = b$, and $x_4, x_5, x_6 = 0$ which would describe the world-surface of such a D4-brane. Note that we can surround it with a small 5-sphere that can be deformed from infinity down to the vertex of the paraboloid without intersecting any charge (and, in particular, without intersecting the D6-brane at the origin of the ALE space). Two-dimensional projections of such spheres lie on the boundaries of the tubes shown in figure 3 below. Thus, for $b \neq 0$, we may discuss conservation of D4-brane charge in direct parallel with section 2. Since there are no branes exiting the vertex of the paraboloid along the $\theta = 0, \pi$ axes, the D4-brane charge at the vertex must vanish by rotational invariance, as can be checked directly from (4.5). It follows that all D4-brane charge may be thought of as generated by the NS5-brane in the background field of the D6-brane through the charge non-conservation laws of type-IIA supergravity.

We would now like to address the issue of the finite $F_{[3]}$ field strength created by the pure multipole configuration of the NS5-brane in the $b \to 0$ limit. The interesting difference between this and, say, point dipoles in familiar Maxwell electrodynamics is that in the familiar Maxwell case such non-trivial pure dipole fields arise only when an infinite amount of charge is present. It is thus of interest to quickly check that the total amount of NS5-brane does not diverge as $b \to 0$, and that it is in fact independent of b. This can be seen from (4.4), but we will show an independent computation in order to further illuminate the geometry of the system.

Define a cube letting $-L \leq x_4, x_5, x_6 \leq L$ with $v = 0$ and $w = w_0$ fixed. A second cube is defined in the same way but with $v = v_0 > b$. We finish closing our 3-manifold with the surface defined by the interval $0 \leq v$ and the faces of the previous cube in (x_4, x_5, x_6).

This 3 surface clearly encloses the 5-brane, and we can easily compute the integral of $F_{[3]}$ over it. To do so first note that, due to (4.8), the integral over the cube at $v = 0$ is zero. The integral over the surface joining the two cubes goes to zero in the limit $L \to \infty$. To see this, let us integrate over one of this surfaces, say, $x_4 = L$,

$$\int_{v=0}^{v_0} \int_{x_5=-L}^{L} \int_{x_6=-L}^{L} AL dx_5 dx_6 dv \,. \tag{4.10}$$

As $L \to \infty$ this integral vanishes as v_0^2/L^2. The charge of the $NS5$-brane is given by the remaining integral, therefore,

$$Q_5 = \frac{1}{NR_{11}} \int_{-\infty}^{\infty} dx_4 \int_{-\infty}^{\infty} dx_5 \int_{-\infty}^{\infty} dx_6 \frac{v_0^2 B}{(x_4^2 + x_5^2 + x_6^2 + v_0^2)^{5/2}} = \frac{Q}{2\pi NR_{11}} \,, \tag{4.11}$$

which is indeed finite and independent of b.

5. Discussion

We have studied a family of type-IIA supergravity solutions obtained by the reduction of a family of 11-dimensional solutions. One member of this family describes a pure multipole configuration of NS5-branes stabilized by the presence of a D6-brane. In this pure-multipole limit, the NS5-brane in fact becomes a D4-brane ending on the D6-brane. By varying a parameter b from ∞ to zero, we obtain a family of BPS solutions in which an NS5-brane moves in from infinity, curves around the D6-brane, and degenerates into the D4-brane ending on the D6-brane. This sort of brane transmutation arises due to the lack of a conservation law for certain types of charge in type-IIA supergravity, so that an NS5-brane in the background field of a D6-brane can act as a source of D4-brane charge. It is clear from the 11-dimensional description that all members of this family preserve the same supersymmetries, so that our family in fact forms a moduli space. By the usual adiabatic arguments, this moduli space can therefore be associated with a dynamical process in which a slowly moving NS5-brane approaches a D6-brane, wraps around it, and transmutes itself into a D4-brane. It is reassuring that our results bear a close resemblance to those of [13], which study a curved test brane (a D5-brane, in that case) in the background of another brane (a D3) using Born-Infeld techniques.

Although we have studied only the near-core solution in explicit form, we expect a corresponding asymptotically flat solution with similar properties. This is indicated, for example, by the results of [14] which extend the Born-Infeld solution of [13] into the asymptotically flat region. One might at first expect that the NS5-brane remains flat far from the D6-brane, in which case the asymptotically flat analogue of our $b = 0$ solution would resemble the diagram. This is a clear analogue of the brane-creation processes discussed by Hanany and Witten [9] and others [10, 11, 12]. As a result, the moduli space mentioned above provides a dynamical supergravity description of such phenomena.

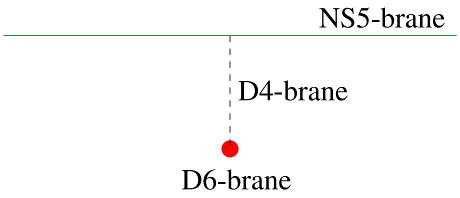

Figure 4: A first guess for the asymptotically flat case.

We recall, however, that when a given brane (our D4-brane) ends on a brane of one dimension higher (the NS5-brane), the larger brane takes the shape of a logarithmic curve and does not quite become flat at infinity [25]. Thus, the figure above will be somewhat modified. In any case, we expect a limit in which the flatter parts of the NS5-brane move up the page and off to infinity so that the total NS5-brane charge vanishes as measured by any gaussian surface in the spacetime. This limiting solution represents a D4-brane ending on a D6-brane in the presence of a nonzero Neveu-Schwarz field strength.

As this solution has three translational symmetries, it is straightforward to T-dualize up to three times, obtaining solutions with multipole NS5-branes representing D3-branes ending on D5-branes, D2-branes ending on D4-branes, or D1-branes ending on D3-branes. Either of the type-IIB solutions are readily S-dualized as well, and this last one then yields a smeared solution containing fundamental strings ending on a D3-brane stabilizing a pure multipole configuration of D5-brane.

Returning to our near-core solutions, one issue that we should pin down more carefully is whether pure-multipole configurations of NS5-brane charge are in fact generically associated with the ending of a D4-brane on a D6-brane or whether there exists another solution in which only the fields usually associated with D4- and D6-branes are excited. That the presence of a nonzero Neveu-Schwarz field *is* generic can be argued by charge conservation. As discussed in section 2, the presence of a D6-brane is not by itself enough to foil conservation of D4-brane charge. Breaking charge conservation necessarily requires the presence of a Ramond-Ramond field as well. However, one might seek a solution in which D4-brane charge *is* conserved but merely flows away along the D6-brane in the x_4, x_5, x_6 directions. Indeed, a striking feature of our solution is that no D4-brane charge leaks into the D6-brane at all.

We therefore turn to a second argument that the NS-field is generic. Let us T-dualize our solution 3 times (to obtain a D-string ending on a D3-brane) and then S-dualize to obtain a fundamental string ending on a D3-brane. T-duality will leave our tightly wrapped NS5-brane an NS5-brane, and S-duality will turn it into a D5-brane. Thus, if an NS5-brane is always associated with a D4-bane ending on a D6-brane, we would expect a fundamental string ending on a D3-brane to be associated with a 'pure-multipole' D5-brane. This brane-ending configuration was investigated in the appendix of [18] using perturbative techniques and the Dirac-Born-Infeld action for the branes. A look at the Ramond-Ramond fields found there does indeed show the presence of a magnetic fields of the sort that would be associated with a certain pure-multipole configuration of D5-branes. Note, however, that the charge non-conservation effect could not be seen in [18] as the fields were computed there only to first order in the charges, while we see from (2.10) that charge non-conservation is a second order effect. Similarly, charge non-conservation will not be seen in other lowest order perturbative calculations.

Finally, a third argument that the solution studied above is in fact 'the' solution for D4-branes ending on D6-branes is obtained by considering what happens when we add a second paraboloid of NS5-branes enclosing the negative axis. This is most easily studied in the simplified example of section 2, but may be done in the full 11-dimensional supergravity case as well. One begins with the spacetime considered above having branes at $|V| = b$ oriented along the W plane. One then inserts an additional set of branes oriented along the V plane and located at $|W| = b$. In the limit $b \to 0$, we obtain two orthogonally intersecting branes at $V = 0$ and $W = 0$. After Kaluza-Klein reduction, we find strings along the positive and negative axes. It is readily checked that the Neveu-Schwarz field strength vanishes when the signs are chosen such that these form a continuous D4-brane passing through the D6-brane. Thus, assembling two of our half-branes does in fact make a familiar Neveu-Schwarz-free D4-brane.

This last argument allows us to easily check that the general solution from section 3, which contains a mixture of D4-, NS5-, and D6-brane charge, is in fact a BPS configuration. The point is that the 11-dimensional configuration of two M5-branes at $V = 0$ and $W = 0$ with signs as above is a typical 'branes at angles' solution (see [26]) preserving 1/4 of the supersymmetries. Now, since the type-IIA configuration with a D4-brane orthogonally intersecting a D6-brane is also a 1/4 BPS solution, we see that no supersymmetries of the branes at angles solution are broken in the reduction process. Since all of these supersymmetries are also present in the case of a single set of branes oriented along the W-plane (located at either $V = 0$ or $|V| = b$), we can indeed be sure that our type-IIA solutions represent a moduli space of BPS configurations preserving the same 1/4 of the supersymmetries.

Having concluded that the $b = 0$ case does indeed represent 'the' solution for a D4-brane ending on a D6-brane, we deduce that another qualitative difference should arise between the near-core solutions studied above and the full asymptotically flat solutions. In our near core solutions, the field strengths $F_{[3]}$ and $\tilde{F}_{[4]}$ associated with NS5- and D4-brane charge are of comparable magnitude. In fact, in an appropriate gauge we have just $\tilde{F}_{[4]} = F_{[3]} \wedge (d\psi/NR_{11} - A_{[1]})$, where NR_{11} is roughly the tension of the D6-brane. In the full asymptotically flat solution for $b = 0$, we would expect the $F_{[3]}$ field to fall off faster than the $\tilde{F}_{[4]}$ field, reducing to that of a pure D4-brane as we move along the D4-brane and away from the D6-brane. Something like this is seen, for example, in the perturbative calculation in [18] of the fields generated by a fundamental string ending on a D3-brane. It appears that the degeneracy seen in the near-core solutions is a consequence of the fact that the near-core region of the Kaluza-Klein monopole has an SO(4) rotational symmetry, while the full solution breaks this to SO(3) $\times$ U(1).

A final natural question to ask is whether all brane ending phenomena in which a 'half-brane' ends on a 'terminal brane' arise in this way. That is, can they all arise as a case of brane transmutation, with the resulting half-branes being formed as some larger brane wraps itself tightly around the terminal brane? If this is the case, then all brane-ending solutions must resemble the one above, in which the charge of the half-brane is in fact absorbed by the terminal brane and does not flow along the terminal brane to infinity. For a fundamental string ending on a D0-brane, it is clear that this is the case, and the Born-Infeld results of [13,14] lead to a similar description of strings stretched between D3-branes and D5-branes. While we cannot reach a definite general conclusion here, we note that the charge non-conservation effect arises from Chern-Simons-like couplings and that an analysis of such couplings as in [27] does make an affirmative answer seem likely. We expect such considerations to provide interesting examples for further study.

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